Processing Data Streams

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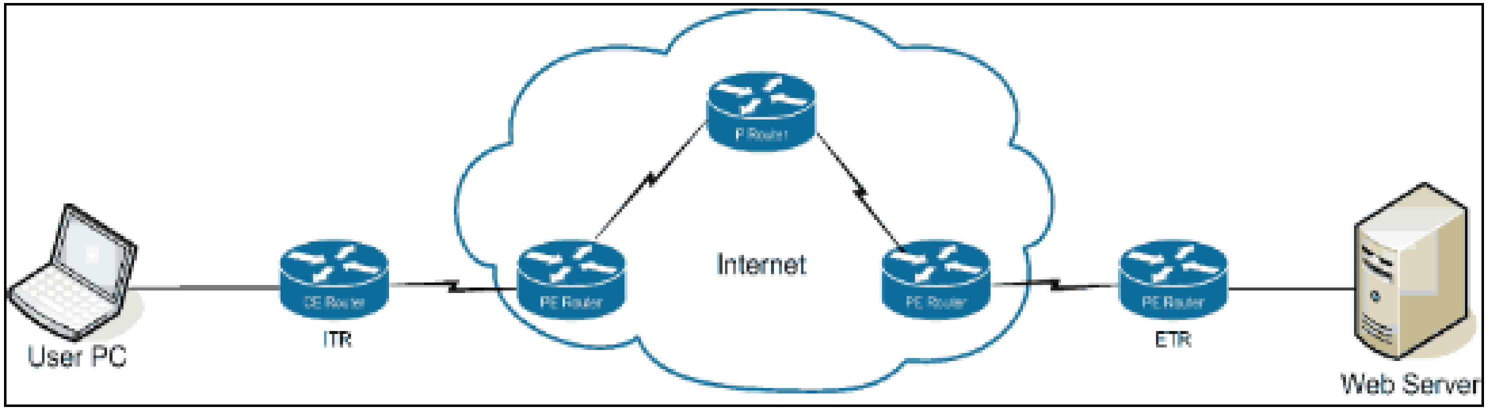
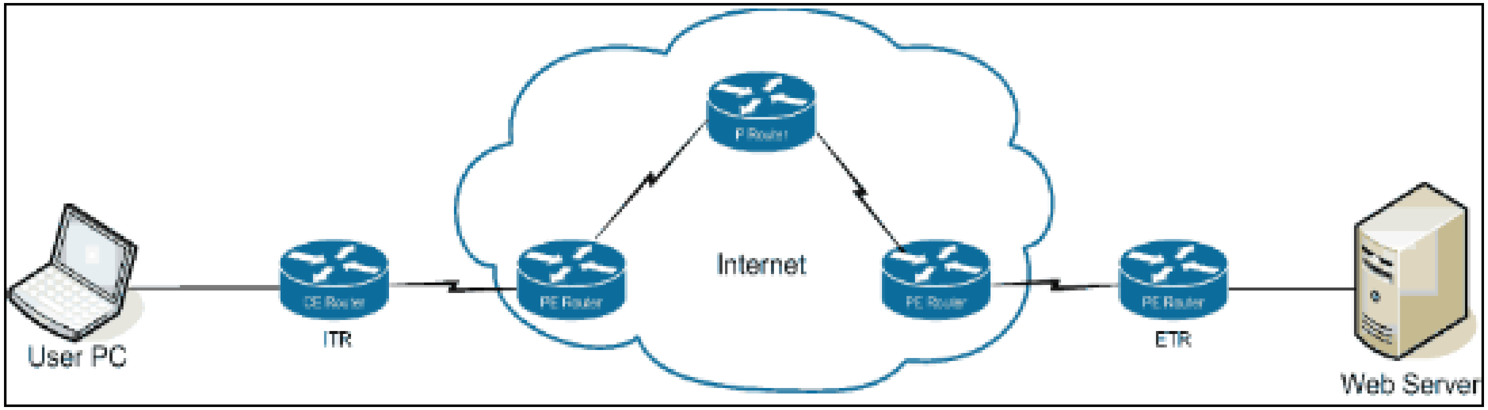
Motivation: Stream Processing

- **In stream processing:**
  - **Data cannot be stored; one-pass**
  - **Analysis needs to be online – no waiting for answers**
  - **Time per update is limited**



Motivation: Stream Processing

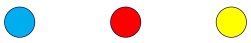
- Many of these trivial questions become extremely difficult for streams
  - How much traffic from/to a certain IP address?
  - How many distinct flows?
  - What are the heavy hitters?



Stream Mining

- **Abstraction:**
  - **Stream is a continuous sequence of *items***

- **Problems:**
  - **Heavy hitters**

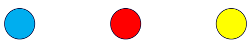


  - **How many distinct items do I have in my stream?**

(6)

  - **Frequent items in the stream**

3 or more:



Stream Mining

- It won't always be possible to give an exact answer
  - Therefore relaxations

- Popular: ε, δ - approximation:
  - In 1- δ of the cases we are at most ε off.

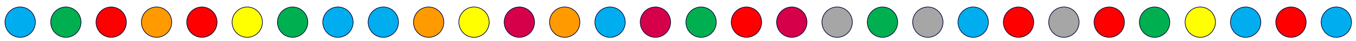
- We will show three examples of stream mining algorithms:
  - Min-wise sampling
  - Number of Distinct Items (min-hash)
  - Frequent items

Outline

- Some Basic Techniques
  - I. Heavy hitters
  - II. Frequent items
- Sketching
  - III. Distinct count sketches
  - IV. Count-Min Sketch
- Semi-streaming:
  - V. Neighborhood function
  - VI. Counting local triangles
- Conclusion

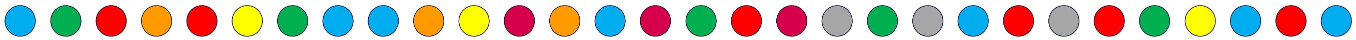
I. Heavy Hitters

- **“Given a stream, identify all items that occur <u>more than</u> 10% of the time”**



I. Heavy Hitters

- “Given a stream, identify all items that occur <u>more than</u> 10% of the time”

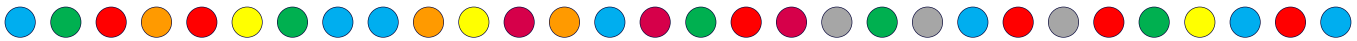


Solution storing 9 colors and counters :

- Summary={}
- For each item  that arrives
  - If (, count) is in Summary:
    update count to count + 1
  - Else if |S|<10:
    add ( , 1) to S
  - Else:
    decrease the count of all pairs in S
    remove all pairs with count = 0

I. Heavy Hitters

- **“Given a stream, identify all items that occur <u>more than</u> 10% of the time”**



Solution storing 9 colors and counters :

- **Guarantee: if an item appeared more than 10% of time, there will be an entry (, count) in the summary**
- **Disadvantage: there may be false positives**
- **Obviously extendible to other thresholds**
  - **Frequency threshold 1/k → k-1 memory places**

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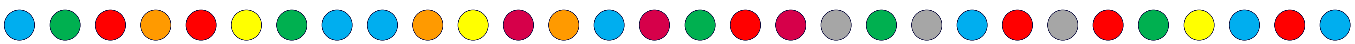
II. Identify Frequent Items

- **Counting every item is impossible**
  - **E.g., all pairs of people that phone to each other**
- **We do not know on beforehand which combinations will be frequent**

- **Example:**



30 items;:8,:6,:5

All others are 3

If frequency is 20%: and need to be outputted

II. Identify Frequent Items

- The following algorithm finds a *superset* of the s-frequent items:
  - Initialization: none of the items has a counter
  - Item ● enters at time t:
    - If ● has a counter: counter(●) ++
    - Else:
      - counter(●) = 1
      - start(●) = t
    - For all other counters ○ do:
      - If counter(○) / (t – start(○) + 1) < s:
        - Delete counter(○), start(○)
- When the frequent items are needed: return all items that have a counter

II. Identify Frequent Items

- **Example: (20%)**

start	# (freq)
1	1 (100%)

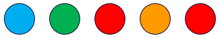
II. Identify Frequent Items

- **Example: (20%)**

	start	# (freq)
	1	1 (50%)
	2	1 (100%)

II. Identify Frequent Items

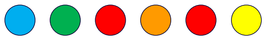
- **Example: (20%)**



	start	# (freq)
	1	1 (20%)
	2	1 (25%)
	3	2 (66%)
	4	1 (50%)

II. Identify Frequent Items

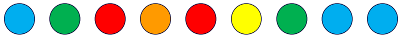
- **Example: (20%)**



	start	# (freq)
	1	1 (17%)
	2	1 (20%)
	3	2 (50%)
	4	1 (33%)
	5	1 (100%)

II. Identify Frequent Items

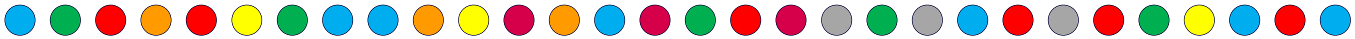
- **Example: (20%)**

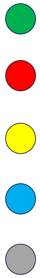


	start	# (freq)
	2	2 (25%)
	3	2 (29%)
	6	1 (25%)
	8	2 (100%)

II. Identify Frequent Items

- **Example: (20%)**



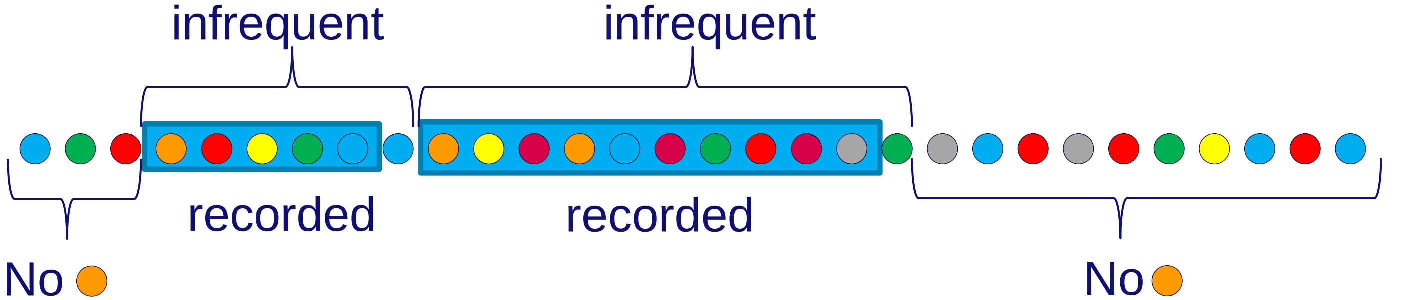


start	# (freq)
2	1 (25%)
17	4 (29%)
27	1 (25%)
8	6 (26%)
19	3 (25%)

II. Identify Frequent Items

- **Why does it work?**
  - **If ● is not recorded, ● is not frequent in the stream**

- **Imagine marking when ● was recorded:**
  - **If ● occurs, recording starts**
  - **Only stopped if ● becomes infrequent since start recording**



- **Whole stream can be partitioned into parts in which ● is not frequent → ● is not frequent in the whole stream**

Algorithm is called "lossy counting"

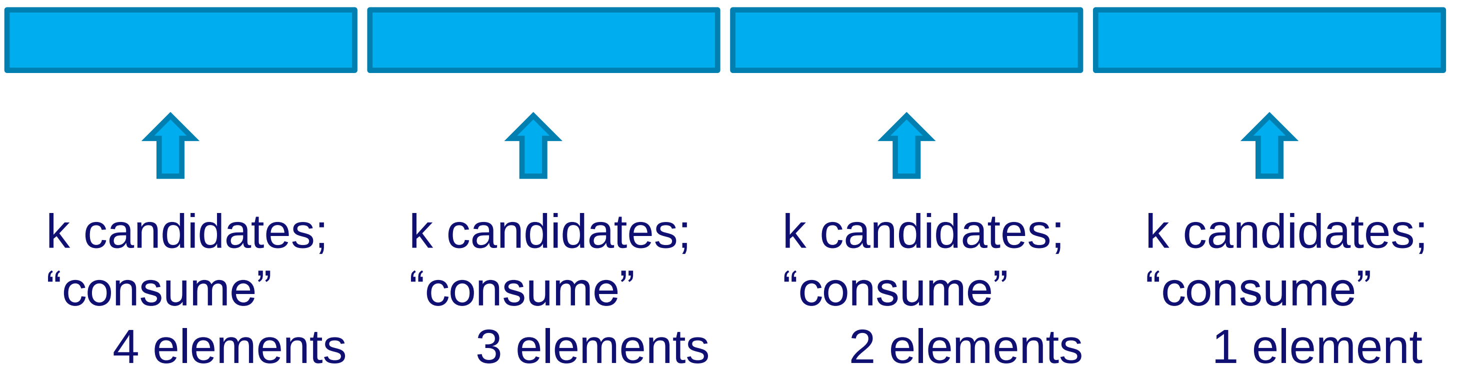
II. Lossy Counting – Space Requirements

- Let N be the length of the stream
- s minimal frequency threshold. Let k=1/s

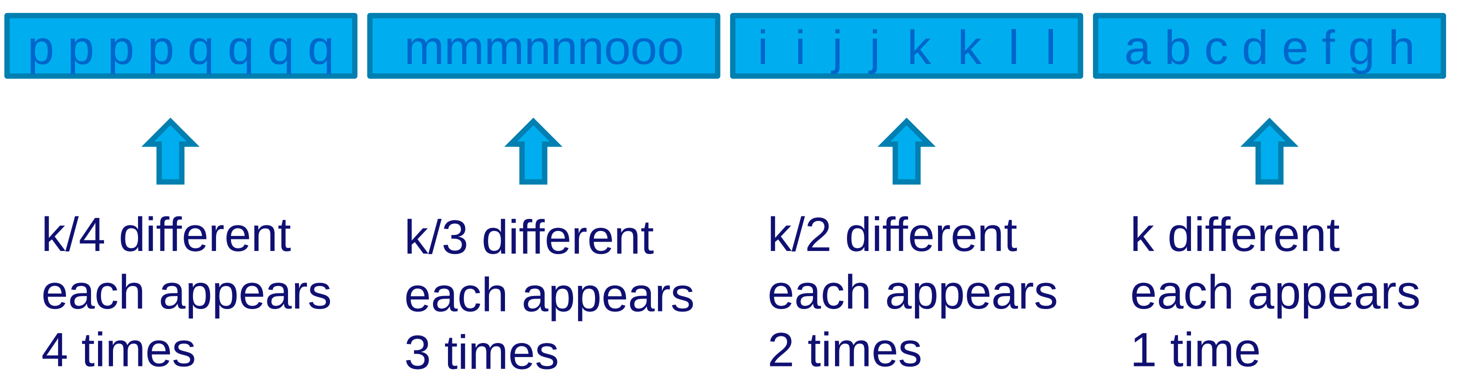
- Item a is in the summary if:
  - a appears once among last k items
  - a appears twice among last 2k items
  - …
  - a appears x times among last xk items
  - …
  - a appears sN times among last N items

II. Lossy Counting – Space Requirements

- **Divide stream in blocks of size k = 1/s**



- **Constellation with maximum number of candidates:**



II. Lossy Counting – Space Requirements

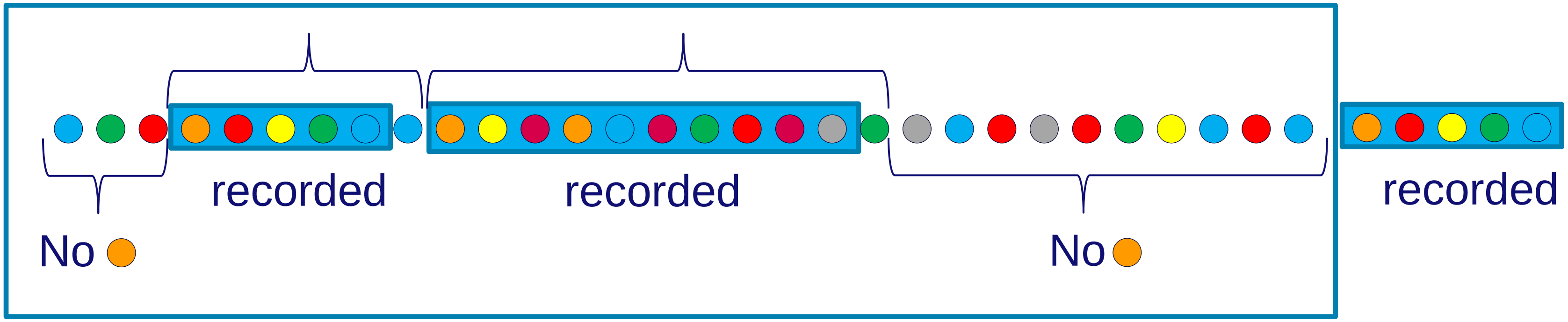
- **Hence total space requirement:**
  $\Sigma_{i=1\ldots N/k}$ k/i ≈ k log(N/k)

- **Recall: k = 1/s**
- **Worst case space requirement: 1/s log(Ns)**

II. Lossy Counting – Guarantee

- **Suppose that we want to know the frequency up to a factor ε**
  - **Same algorithm, yet use ε as minimum support threshold**
  - **Report all items with count ≥ (s- ε) N**

- **Guaranteed: true frequency in the interval**
  [count/N, count/N+ε]



Less than εN occurrences of

II. Lossy Counting - Summary

- **Worst case space consumption:**

  $1/\varepsilon \log(N\varepsilon)$

- **Guarantee: with 100% certainty, the relative error for all s-frequent itemsets is ε**

- **Performs very well in practice**
  - **Optimization: check if item is frequent only every $1/\varepsilon$ steps**

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III. How Many Different Items do I have?

- Number of distinct items is too big to keep all in memory

- Observation:

  If h(.) is a hash function: every $x_i \rightarrow [0,1]$

  Maintain $\min\{ h(x_1), h(x_2), \ldots, h(x_n) \}$

  $E[\min \{ h(x_1), h(x_2), \ldots, h(x_n) \}] = 1/(1+D)$

  with $D = | \{ x_1, x_2, \ldots, x_n \} |$

- Average over many (independent) h to decrease variance
- Called: min-hash algorithm

III. How Many Different Items do I have?

- **Example:**

.13 .25 .17 .85 .33 .52 .13 .25 .17 .85 .33 .52 .33 .52 .13

- **Min h(x) = .13**
- **Estimate D: 1/(1+d) = 0.13 ➔ d = 1/0.13 – 1 ≈ 6.7**

- **Averaging over independent trials makes the result more accurate**

III. How Many Different Items do I have?

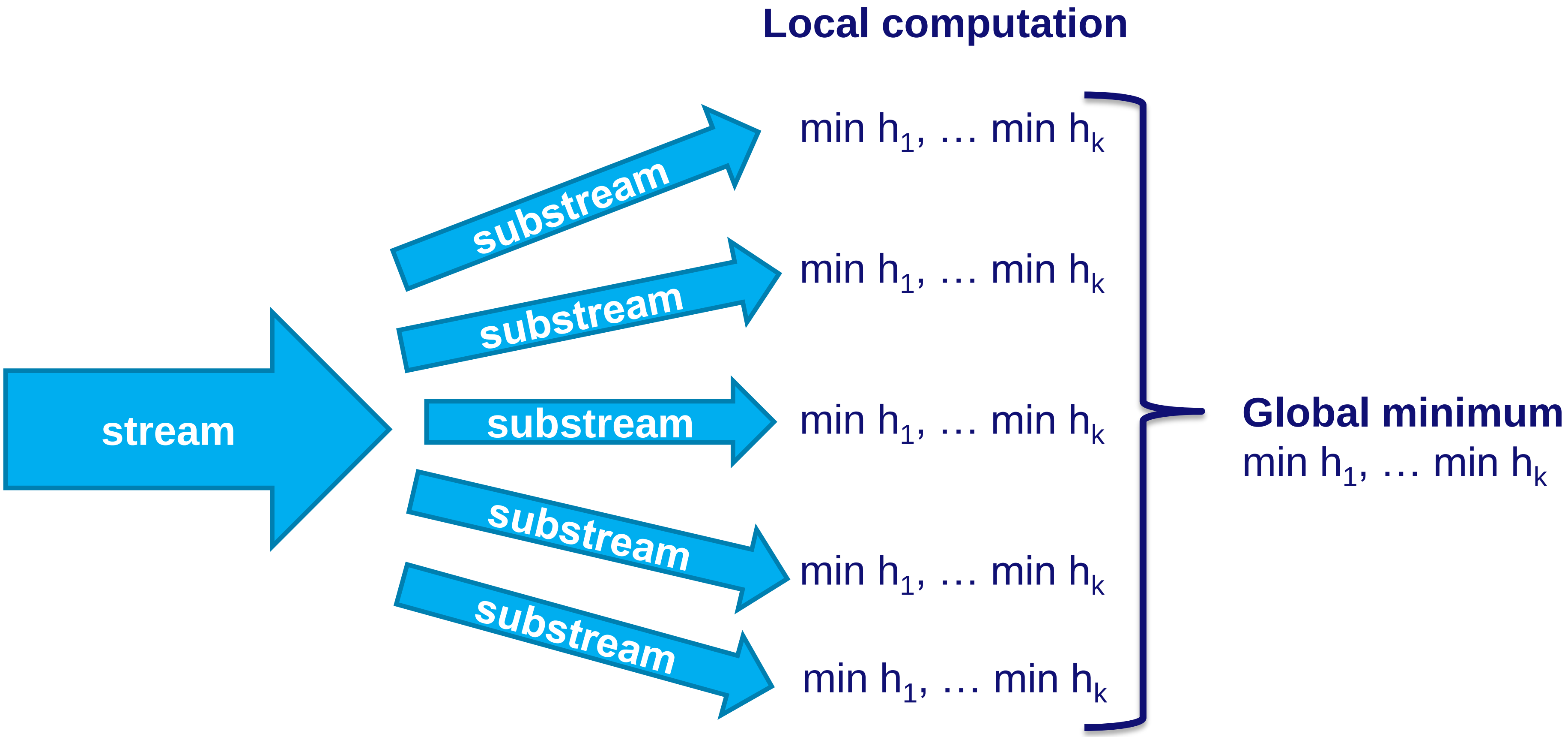
- Many variations on the same idea

- Multiple hash-functions h_1 … h_k
  - H_1 → estimate 1 mean D high variance
  - H_2 → estimate 2 mean D high variance
  - …
  - H_k → estimate k mean D high variance
  - Median {$estimate_i$} mean D low variance

- HyperLogLog sketch: count 1,000,000,000 items with 2% error → 1.5kB

Stream still too fast?

- **No problem; easily parallelizable**
  - **min (min(A), min(B)) = min(A∪B)**



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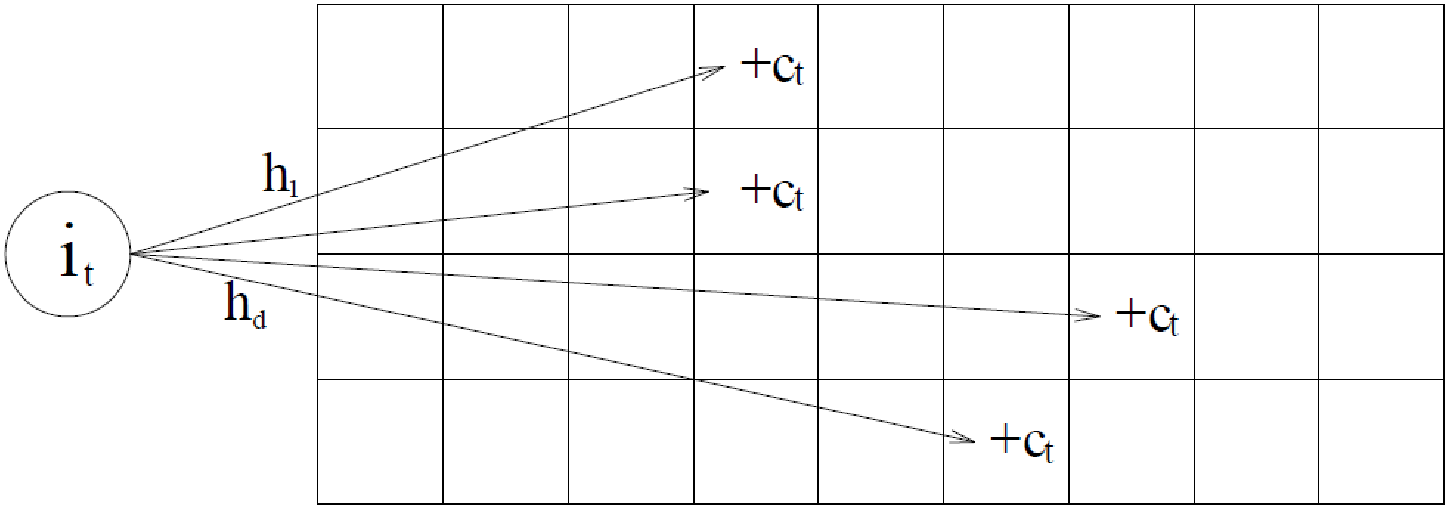
IV. Sketching

- Extension of the model
  - Items + numbers
    - (a,5) → add 5 to a
    - (a,-3) → subtract 3 from the count of item a
  - Query
    - Sum for item a

- New technique based upon a *sketch*
  - Smart summary of the data

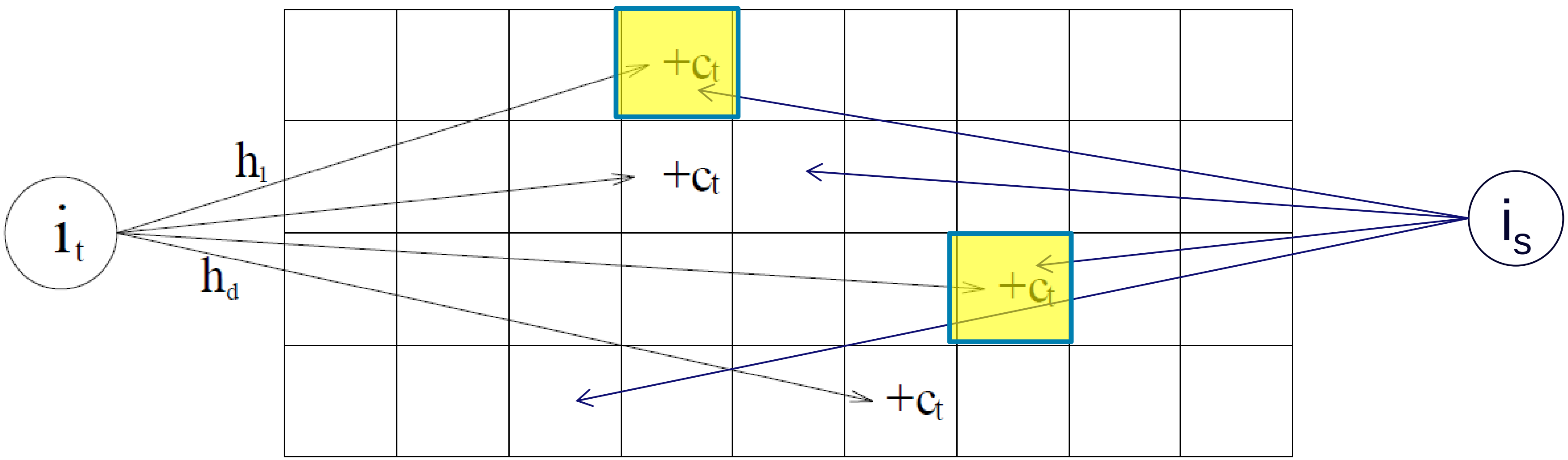
IV. Sketching

- There is not enough space to store sums for all items
- Instead we will store a (*d* x *n*) – matrix S
  - We have d hash functions $h_1, \dots , h_d$
  - The counts of item i_t are stored in cells $S[1,h_1(i_t)], \dots , S[d,h_d(i_t)]$



IV. Sketching

- **Notice that there will be collisions:**



- **For the non-negative case:**
  - **all cells $S[1,h_1(i_t)]$, … , $S[d,h_d(i_t)]$ will be overestimations of the count of i_t**
  - **Return *min*($S[1,h_1(i_t)]$, … , $S[d,h_d(i_t)]$)**

Example: Count-min Sketch

- **CM-Sketch with 3 columns and 4 rows**

h1	0	0	0
h2	0	0	0
h3	0	0	0
h4	0	0	0

- **Stream:**

Example: Count-min Sketch

- **CM-Sketch with 3 columns and 4 rows**

h1	0	0	1
h2	0	0	1
h3	1	0	0
h4	1	0	0

- **Stream:**

Example: Count-min Sketch

- **CM-Sketch with 3 columns and 4 rows**

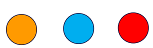
h1	1	0	1
h2	0	1	1
h3	1	0	1
h4	2	0	0

- **Stream:**

Example: Count-min Sketch

- **CM-Sketch with 3 columns and 4 rows**

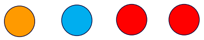
h1	2	0	1
h2	1	1	1
h3	1	1	1
h4	2	0	1

- **Stream:** 

Example: Count-min Sketch

- **CM-Sketch with 3 columns and 4 rows**

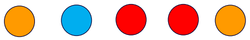
h1	3	0	1
h2	2	1	1
h3	1	2	1
h4	2	0	2

- **Stream:** 

Example: Count-min Sketch

- **CM-Sketch with 3 columns and 4 rows**

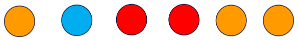
h1	3	0	2
h2	2	1	2
h3	2	2	1
h4	3	0	2

- **Stream:** 

Example: Count-min Sketch

- **CM-Sketch with 3 columns and 4 rows**

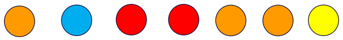
h1	3	0	3
h2	2	1	3
h3	3	2	1
h4	4	0	2

- **Stream:** 

Example: Count-min Sketch

- **CM-Sketch with 3 columns and 4 rows**

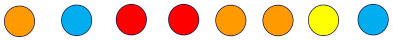
h1	3	0	4
h2	3	1	3
h3	3	3	1
h4	4	1	2

- **Stream:** 

Example: Count-min Sketch

- **CM-Sketch with 3 columns and 4 rows**

h1	4	0	4
h2	3	2	3
h3	3	3	2
h4	5	1	2

- **Stream:** 

Example: Count-min Sketch

- **CM-Sketch with 3 columns and 4 rows**

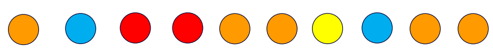
h1	4	0	5
h2	3	2	4
h3	4	3	2
h4	6	1	2

- **Stream:** 

Example: Count-min Sketch

- **CM-Sketch with 3 columns and 4 rows**

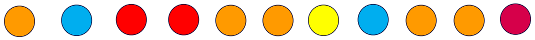
h1	4	0	6
h2	3	2	5
h3	5	3	2
h4	7	1	2

- **Stream:** 

Example: Count-min Sketch

- **CM-Sketch with 3 columns and 4 rows**

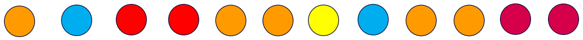
h1	4	1	6
h2	3	2	6
h3	6	3	2
h4	7	2	2

- **Stream:** 

Example: Count-min Sketch

- **CM-Sketch with 3 columns and 4 rows**

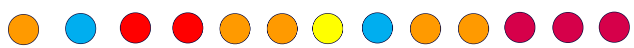
h1	4	2	6
h2	3	2	7
h3	7	3	2
h4	7	3	2

- **Stream:** 

Example: Count-min Sketch

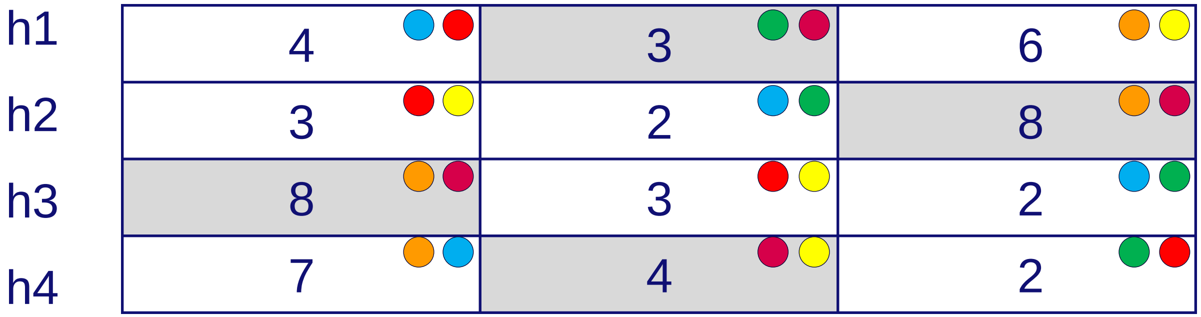
- **CM-Sketch with 3 columns and 4 rows**

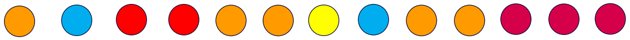
h1	4	3	6
h2	3	2	8
h3	8	3	2
h4	7	4	2

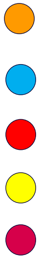
- **Stream:** 

Example: Count-min Sketch

- **CM-Sketch with 3 columns and 4 rows**



- **Stream:** 
- **Report frequencies:**



estimate	**true count**
6	5
2	2
2	2
3	1
3	3

IV. Sketching

- **Usually for many more items than in the example**
  - **Number of items usually exceeds number of cells by orders of magnitude**
- **Especially effective if only few “heavy” items, many rare items**
  - **E.g., Zipfian distribution**

- **Tight guarantees on the estimation**
  $w = \lceil \frac{e}{\varepsilon} \rceil$ and $d = \lceil \ln \frac{1}{\delta} \rceil$; **$h_1,..,h_d$ pairwise independent with probability** $1-\delta$ **,** $\hat{a}_i \leq a_i + \varepsilon ||\boldsymbol{a}||_1$

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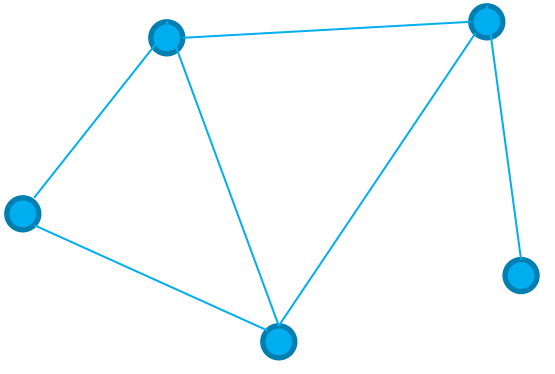
V. Neighborhood Function

- **Count the number of pairs of nodes at distance 1, 2, 3, …**



1: 6
2: 3
3: 1

- **Important statistics; allows to compute average degree, diameter, effective diameter.**

V. Neighborhood Function

- **Straightforward algorithm**

Set $N_0(v) = \{v\}$

For i = 1 to r:

For all v in V:

$N_i(v)=N_{i-1}(v)$

For {v,w} in E:

$N_i(v) \leftarrow N_i(v) \cup N_{i-1}(w)$

$N_i(w) \leftarrow N_i(w) \cup N_{i-1}(v)$

Return $avg(|N_1(v)|), avg(|N_2(v)|-|N_1(v)|), \ldots$

- **Time: $O(r |V| |E|)$**
- **Space: $O(|V|^2)$**

V. Neighborhood Function

- Observation: we can replace every set by a *summary*
  - Take union, cardinality, add an element

- Size of set: V versus size of summary: k <<< |V|
  - |V| versus log(log(|V|))
  - With the summary we can:

- Time O(r k |E|)
- Space O(k |V|)

- Speedup is enormous (1000s of times faster!)

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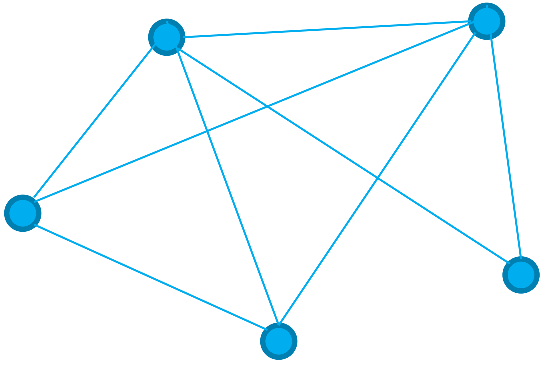
VI. Streaming Graph Processing

- **Example of an application of stream processing for attacking a truly big data problem**



- **Given a graph, count, for every node, in how many triangles it appears**

Becchetti et al. Efficient Semi-streaming algorithms for local triangle counting in massive graphs. In: KDD'08

VI. Streaming Graph Processing

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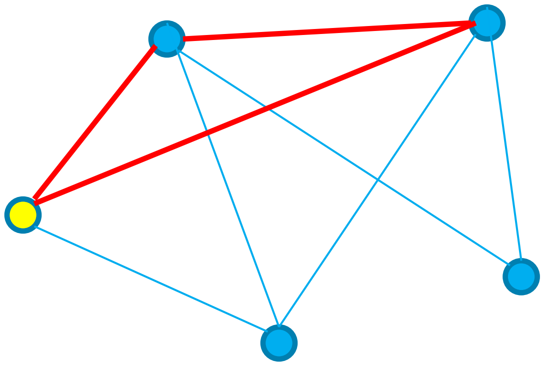

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VI. Streaming Graph Processing

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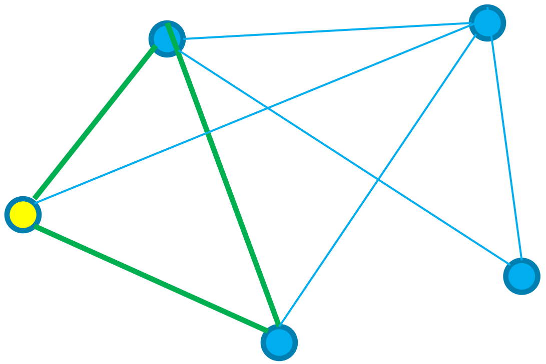

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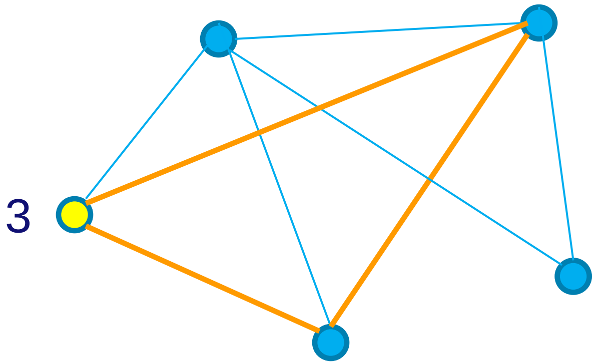

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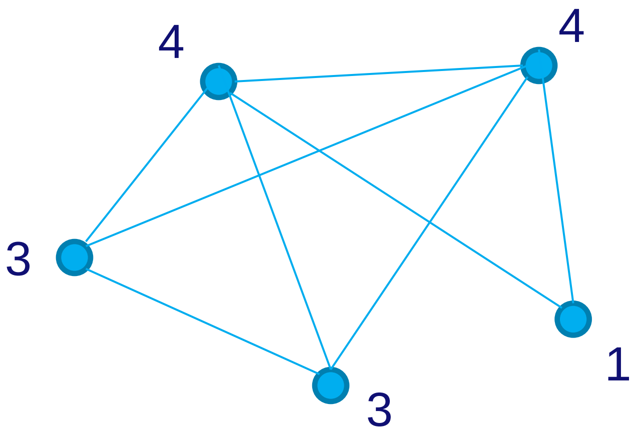
VI. Streaming Graph Processing

- **Example of an application of stream processing for attacking a truly big data problem**

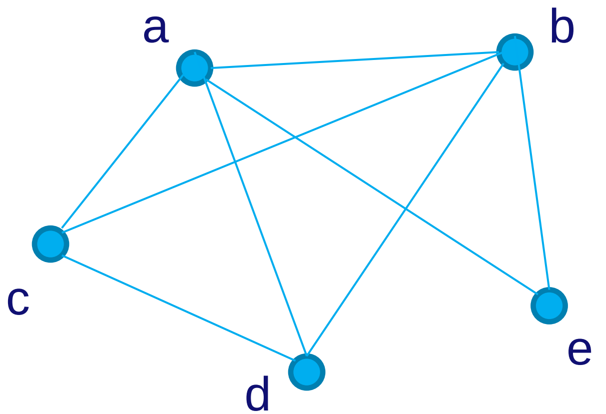


- **Given a graph, count, for every node, in how many triangles it appears**
  - **Indicator for connectedness of the node into the community**

VI. Storage Model

- **Graph stored as a stream of edges**

src	dest
a	b
a	c
a	d
a	e
b	c
b	d
b	e
c	d

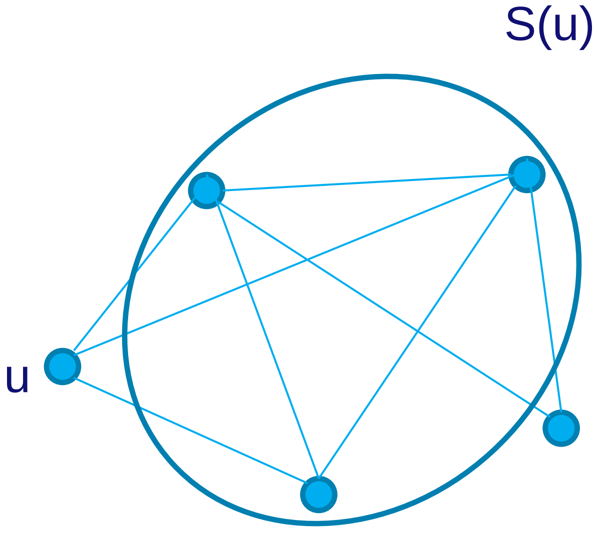


- **Random access is *expensive***
- **Access data using limited number of linear scans**

VI. Counting Triangles - Notation

- **S(u) : neighbors of u**
- **T(u) : number of triangles in which u is involved**
- **d_u : degree of u**
- **Local clustering coefficient:**

$$\frac{2\,T(u)}{d_u(d_u-1)}$$



WHY counting triangles? T(u) and local clustering coefficient are informative features for many problems

VI. Counting Triangles

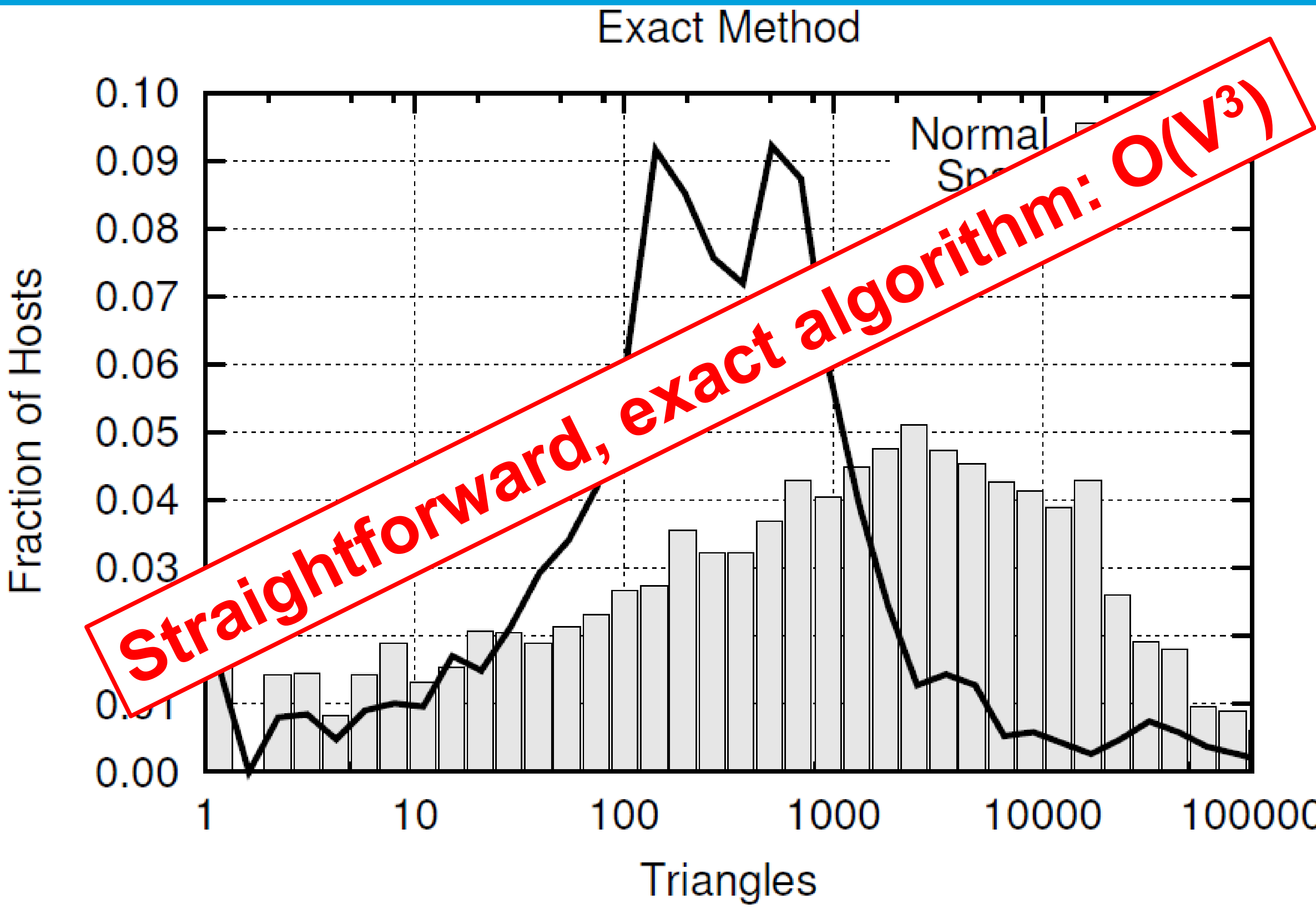
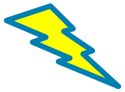


Figure from: Becchetti et al. Efficient Semi-streaming algorithms for local triangle counting in massive graphs. In: KDD'08

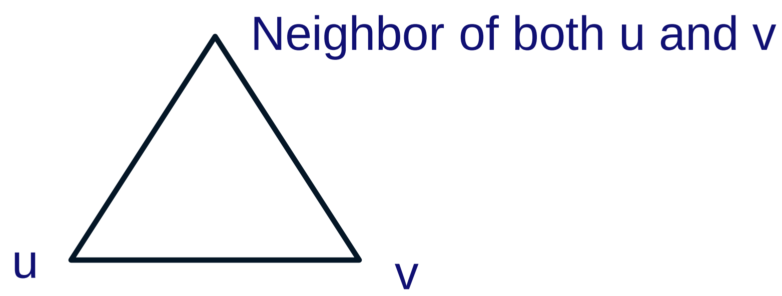
VI. We Need Brains, Not Just More Power …

- **N processors can speed up only a factor N *at most***
  - **So, for N nodes, we need N^2 processors to make it linear** 

- **Solution will be based upon:**

$$T(u) = \sum_{v \in S(u)} |S(u) \cap S(v)| / 2$$

and a smart way to do intersection *approximately*



- **Building block: estimate for the “Jaccard coefficient”**

VI. Brute Force- Example

1. **Compute**

   S(a) = {b,c,d,e}
   S(b) = {a,c,d,e}
   S(c) = {a,b,d}
   S(d) = {a,b,c}
   S(e) = {a,b}

Too big to fit
into memory

2. **Initialize all T(u) to 0**
3. **Iterate over all edges (u,v)**

   Add |S(u) ∩ S(v)| to T(u) and T(v)
4. **Divide all T(u) by 2**

Random access
to secondary storage

src	dest
a	b
a	c
a	d
a	e
b	c
b	d
b	e
c	d

VI. Building Block: Jaccard Coefficient

$$J(A,B) = \frac{|A \cap B|}{|A \cup B|}$$

Indicates how similar the sets A and B are.

Example:

$J(\{a,b,c\},\{c,d\}) = 1/4$

$J(\{a,b,c\},\{b,c,d\}) = 2/4$

Used, e.g., to detect near duplicates (Altavista)

A set of n-grams in document 1

B set of n-grams in document 2

VI. Building Block: Jaccard Coefficient

Let A, B be subsets of U

h is a function mapping elements of U to {1,2,…,|U|}

Example: d → 1, c→ 2, a → 3, b → 4

Let $\min_h(A) := \min_{a \in A} h(a)$

$$\Pr[\ \min_h(A) = \min_h(B)\ ]$$

$$= \Pr[\ \text{min of all elements in } A \cup B \text{ is in } A \cap B\ ]$$

$$= |A \cap B| \,/\, |A \cup B|$$

$$= J(A,B)$$

VI. Building Block: Jaccard Coefficient

For random h, Pr[$\min_h(A) = \min_h(B)$] = J(A,B)

"estimate" this probability by sampling many independent h

➔ excellent estimate of J(A,B)

$|A \cap B| = J(A,B)\ |A \cup B| = J(A,B)\ (|A|+|B|-|A \cap B|)$

$= (|A| + |B|)\ J(A,B)\ /\ (1+J(A,B))$

VI. Building Block: Jaccard Coefficient

- **Independent functions $h_1, \ldots, h_m$**
- **"signature" of set A:**

$|A|$ and vector ($\min_{h1}(A), \min_{h2}(A), \ldots, \min_{hm}(A)$)

- **Estimating $| A \cap B |$**
  - **$(a_1, \ldots, a_m)$ vector for A**
  - **$(b_1, \ldots, b_m)$ vector for B**

Let $e = \# \{ i \mid a_i = b_i \}$

e / m is an estimator for $J(A,B)$

$| A \cap B | \approx (|A| + |B|) \, e / (m + e)$

VI. Building Block: Jaccard Coefficient

Example: **U = { a, b, c, d, e }**

	h_1	h_2	h_3	h_4
a	1	2	5	2
b	2	5	2	4
c	3	1	4	5
d	4	4	1	3
e	5	3	3	1

A = { a, b }

B = { b, c, d }

C = { a, b, c, e }

A	1	2	2	2
B	2	1	1	3
C	1	1	2	1

J(A,B) = 1/4 ; estimate: 0 → 0

J(A,C) = 1/2 ; estimate: 1/2 → 6 x 2/6 = 2

J(B,C) = 2/5 ; estimate: 1/4 → 7 x 1/5 = 7/5

VI. The Algorithm

- **Memory requirements:**
  - **Main memory: couple of bytes per vertex**
  - **External memory: One entry for every edge e**

- **Based upon $T(u) = \sum_{v \in S(u)} |S(u) \cap S(v)| / 2$**
  - **For every edge (u,v) we maintain estimate of $|S(u) \cap S(v)|$ in external memory**
    - **Using m functions $h_1, h_2, \ldots, h_m$**

VI. Intelligent Intersection Algorithm - Example

1. **Compute**

   $Sig(a) = (a_1,\ldots,a_m)$
   $Sig(b) = (b_1,\ldots,b_m)$
   $Sig(c) = (c_1,\ldots,c_m)$
   $Sig(d) = (d_1,\ldots,d_m)$
   $Sig(e) = (e_1,\ldots,e_m)$

Still quite expensive on memory

src	dest
a	b
a	c
a	d
a	e
b	c
b	d
b	e
c	d

2. **Initialize all T(u) to 0**
3. **Iterate over all edges (u,v)**

   Compute $e = \#\{ i \mid u_i = v_i \}$

   Estimate $|S(u) \cap S(v)|$ based upon e

   Add estimate of $|S(u) \cap S(v)|$ to T(u) and T(v)
4. **Divide all T(u) by 2**

VI. Intelligent Intersection Algorithm - Example

For p = 1 to m:

1. **Compute**

   Sig(a) = h_p(S(a))

   …

   Sig(e) = h_p(S(e))

2. **Iterate over all edges (u,v)**

   If p==1: initialize Z_{uv} to 0

   If h_p(u) == h_p(v): add 1 to Z_{uv}

Iterate over all Z_{uv}:

Estimate |S(u) ∩ S(v)| based upon Z_{uv}

Add estimate of |S(u) ∩ S(v)| to T(u) and T(v)

Divide all T(u) by 2

src	dest
a	b
a	c
a	d
a	e
b	c
b	d
b	e
c	d

VI. The Complete Algorithm

```
for p : 1 to m
        for every vertex v
                min(v) := ∞
        for every edge (v,w)
                min(v) := min(min(v) , h_p(w))
                min(w) := min(min(w) , h_p(v))
        for every edge (v,w)
                if p==1 then Z_{v,w} := 0
                if min(v) == min(w) then
                        Z_{v,w} := Z_{v,w} + 1
for every Z_{v,w} :
        T(v) := T(v) + estimate of |S(v) ∩ S(w)|
        T(w) := T(w) + estimate of |S(v) ∩ S(w)|
for all vertices v:
        T(v) := T(v)/2
```

VI. The Complete Algorithm

for p : 1 to m

for every vertex v

min(v) := ∞

for every edge (v,w)

min(v) := min(min(v) , h_p(w))

min(w) := min(min(w) , h_p(v))

for every edge (v,w)

if p==1 then $Z_{v,w}$:= 0

if min(v) == min(w) then

$Z_{v,w} := Z_{v,w} + 1$

for every $Z_{v,w}$:

T(v) := T(v) + estimate of |S(v) ∩ S(w)|

T(w) := T(w) + estimate of |S(v) ∩ S(w)|

for all vertices v:

T(v) := T(v)/2

In memory

Sequential read

Sequential write

Secondary storage

min(u) for all vertices u: in memory
T(u) for all vertices: in memory
$Z_{u,v}$ for all edges (u,v): on disk

VI. Counting Triangles

- **Reduce complexity from $|V|^3$ to O(m|E|)**
- **Computing power is great, but only gives you an *at most linear speed-up***
- **Willingness to sacrifice exactness leads to incredible performance gains**
- **Resulting approximation still excellent feature**

Outline

- Some Basic Techniques
  - I. Heavy hitters
  - II. Frequent items
- Sketching
  - III. Distinct count sketches
  - IV. Count-Min Sketch
- Semi-streaming:
  - V. Neighborhood function
  - VI. Counting local triangles
- Conclusion

Conclusion

- Stream mining:
  - Severe computational restrictions
  - Yet, surprisingly many operations are still possible
    - Heavy hitters
    - Number of distinct items
    - Frequent items
    - “Cash register”

- Counting triangles and neighborhood function as applications